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FIXED POINT THEOREM FOR SET-VALUED RESTRICTED-QUASI-CONTRACTION MAPS IN A D^*- METRIC SPACE

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ABSTRACT

In this paper we achieve a fixed point theorem for D^*-metric set-valued restricted-quasi-contraction mappings in a D^*-metric space. The result was obtained analogously by the method followed by Aydi [1] and extends the set-valued fixed point theory from b-metric spaces to D^*-metric spaces.

KEYWORDS: Fixed Point Theorem, Quasi-Contraction Mappings, Set-Valued Mappings, D^*-Metric Space, D^*-Metric Set-Valued Restricted, Quasi, Contraction Mappings

1. INTRODUCTION AND PRELIMINARIES

The idea of the b-metric space as a generalization of the notion of an ordinary metric space was defined in 1998 by Czerwik, his motivation for this new concept was an observation that some mathematical problems lead to a different kind of triangle inequality in which the ordinary triangle inequality does not hold see Example 1.1 and [2]**.**

The D-metric space was introduced by B. C. Dhage [3] as an attempt to obtain equivalent metric space results in a more general setting. The geometric representation of the D-metric space can be thought of as the perimeter of a triangle in $\mathbb{R}^2$ with vertices x, y and z.

Unfortunately, many authors such as Mustafa, Sims and Naidu have found flaws with the topological properties of the D-metric space see [4, 5], which lead Sedghi [6] to provide a modification of the D-metric space known as the D^*-metric space.

Definition 1.1 [2] Let$X$ be a non-empty set and $s \geq 1$ be a given real number. A function $d: X \times X \longrightarrow \mathbb{R}$, is called a b-metric if for all $x, y, z \in X$ the following conditions are satisfied,

- $d(x, y) \geq 0$
- $d(x, y) = 0$ if and only if $x = y$
- $d(x, y) = d(y, x)$
- $d(x, y) \leq s\big(d(x, z) + d(z, y)\big)$.

The pair $(X, d)$is called a b-metric space.

It is clear that the (usual) d-metric space is a b-metric space, however the converse is not true. The following example shows that a b-metric on X need not be a metric on X.

Example 1.1: Let $X = \{x_1, x_2, x_3\}$, $d(x_1, x_3) = k \geq 2, d(x_1, x_2) = d(x_2, x_3) = 1$, $d(x_i, x_j) = d(x_j, x_i)$ for $i, j = 1,2,3$, $i \neq j$ and $d(x_i, x_i) = 0$ for $i = 1,2,3$.

Then, $d(x_i, x_j) \leq \frac{k}{2}\left[d(x_i, x_m) + d(x_m, x_j)\right]$for $m, i, j = 1,2,3$.

Clearly (X, d) is a b-metric space. However for $k > 2$(say$k = 3$)

l$d(x_1, x_3) = 3$, $d(x_1, x_2) = 1$, $d(x_2, x_3) = 1$

This implies, $d(x_1, x_3) \nleq d(x_1, x_2) + d(x_2, x_3)$. Hence the ordinary triangle inequality does not hold. Therefore (X, d) is not a metric space.

Definition 1.2 [3] Let$X$ be a non-empty set and let $D: X \times X \times X \longrightarrow [0, \infty)$ be a function satisfying the following conditions, for all $a, x, y, z \in X$

- $D(x, y, z) \geq 0$
- $D(x, y, z) = 0$ if and only if $x = y = z$
- $D(x, y, z) = D(x, z, y) = D(y, x, z) = D(y, z, x) = D(z, x, y) = D(z, y, x)$
- $D(x, y, z) \leq D(x, y, a) + D(x, a, z) + D(a, y, z)$.

Then D is called a D-metric on X and the pair (X, D) is called D-metric space.

In addition, if $D(x, x, y) = D(x, y, y)$ for all $x, y \in X$ then D is called a symmetric D-metric.

It should be noted that the usual d-metric is often called the distance function while the D-metric is called the diameter function.

Definition 1.3 [6] Let$X$ be a non-empty set and let $D^*: X \times X \times X \longrightarrow [0, \infty)$ be a function satisfying the following conditions, for all $a, x, y, z \in X$

- $D^*(x, y, z) \geq 0$
- $D^*(x, y, z) = 0$ if and only if $x = y = z$
- $D^*(x, y, z) = D^*(x, z, y) = D^*(y, x, z) = D^*(y, z, x) = D^*(z, x, y) = D^*(z, y, x)$
- $D^*(x, y, z) \leq D^*(x, y, a) + D^*(a, z, z)$.

Then D^* is called a D^*-metric on X and the pair (X, D^*) is called D^*-metric space.

Remark 1.1 [6]Let(X, D^*) be a D^*-metric space.

- Then $D^*(x, y, y) = D^*(x, x, y)$ for all $x, y \in X$.
- From (a) we can write condition (iv) of Definition 1.3 as

$D^*(x, y, z) \leq$ a $D^*(x, a, a) +$ a $D^*(a, y, z)$for all $x, y, z, a \in X$

Definition 1.4 [6]A sequence $\{x_n\}$ in a D^*-metric space is said to be convergent or D^*-convergent to $x \in X$ if for every $\epsilon > 0$, there exist $n_0 = n_0(\epsilon) \in \mathbb{N}$, such that$D^*(x_n, x_n, x) < \in$ for all $n > n_0$. That is,

$D^*(x_n, x_n, x) \longrightarrow 0$ as $n \longrightarrow \infty$, or simply $\lim_{n\to\infty} x_n = x$.

It is also noted that $D^*(x_n, x_n, x) = D^*(x, x, x_n)$for all $n > n_0$.

Definition 1.5 [6] A sequence $\{x_n\}$ in a D^*-metric space is said to be a D^*-cauchy sequence if for every $\epsilon > 0$, there exist $n_0 = n_0(\epsilon) \in \mathbb{N}$, such that$D^*(x_n, x_n, x_m) < \in$ for all $m, n > n_0$.

The space (X, D^*) is said to be complete if every D^*-Cauchy sequence in X is convergent in X.

Definition 1.6 Let (X, d) be a metric space. The family of all non-empty closed and bounded subsets of X is denoted by $CB(X)$.

The following definition is identical to the definition given by Ashraf [7]

Definition 1.7 Let (X, d) be a metric space, (X, D) be a D-metric space and $CB_D(X)$ be the family of all non-empty closed and bounded subsets of X in a D-metric space.

- The distance between any point $x \in X$ and any two non-empty subsets $A, B \in CB_D(X)$ is denoted by $D(x, A, B)$ and is defined by,

  $D(x, A, B) = d(x, A) + d(x, B) + d(A, B)$

  Where,

  $d(x, A) = \inf\{d(x, y): y \in A\}$,$d(x, B) = \inf\{d(x, y): y \in B\}$and $d(A, B) = \inf\{d(a, b): a \in A, b \in B\}$.

- Let $A, B, C \in CB_D(X)$. The Hausdorff D-metric or Hausdorff D-metric distance is denoted by $D_H(A, B, C)$ and is defined by,

  $D_H(A, B, C) = \max\{sup_{x \in A} D(x, B, C), sup_{x \in B} D(x, C, A), sup_{x \in C} D(x, A, B)\}$.

Informally, if (X, D) is a D-metric space, the D-metric Hausdorff distance is the greatest of all distances from a point in one set to the closest points in the other two sets plus the distance between those two sets.

Definition 1.8 [8]Let(X, d) be a metric space. The set-valued map $T: X \longrightarrow CB(X)$ is said to be a q-set-valued quasi-contraction if,

$d_H(Tx, Ty) \leq q.max\{d(x, y), d(x, Tx), d(y, Ty), d(x, Ty), d(y, Tx)\}$, for any $x, y \in X$ where $0 \leq q < 1$ and d_H denotes the Hausdorff metric on $CB(X)$ induced by d. That is,

$d_H(A, B) = max\{sup_{x \in A} d(x, B), sup_{y \in B} d(y, A)\}$for all $A, B \in CB(X)$.

The following is a fixed point theorem for set-valued quasi-contraction maps in b-metric spaces.

Theorem 1.1 [1]Let(X, d) be a complete b-metric space. Suppose that T is a q-set-valued quasi-contraction. Assume that $< \frac{1}{s^2+s}$, then T has a fixed point in X, that is there exist $u \in X$ such that $u = Tu$ $(u \in Tu)$.

Our main result extends Theorem 1.1 to D^*-metric spaces.

MAIN RESULTS

Before we get into the specifics of the main result, the following definitions are required. It shows the natural extension of Definition 1.6 and Definition 1.7 as it applies to the D^*-metric space.

Definition 2.1 Let (X, D^*) be a D^*-metric space. The family of all non-empty closed and bounded subsets of X is denoted by $CB_{D^*}(X)$.

Definition 2.2 Let (X, d) be a metric space, (X, D^*) be a D^*-metric space and $CB_{D^*}(X)$ be the family of all non-empty closed and bounded subsets of X in a D^*-metric space.

- The distance between any point $x \in X$ and any two non-empty subsets $A, B \in CB_{D^*}(X)$ is denoted by $D^*(x, A, B)$ and is defined by,

  $D^*(x, A, B) = d(x, A) + d(x, B) + d(A, B)$

  Where, $d(x, A) = \inf\{d(x, y): y \in A\}$, $d(x, B) = \inf\{d(x, y): y \in B\}$ and $d(A, B) = \inf\{d(a, b): a \in A, b \in B\}$.

- Let $A, B, C \in CB_{D^*}(X)$. The Hausdorff D^*-metric or Hausdorff D^*-metric distance is denoted by $D^*{}_H(A, B, C)$ and is defined by,

  $D^*{}_H(A, B, C) = \max\{sup_{x \in A} D^*(x, B, C), sup_{x \in B} D^*(x, C, A), sup_{x \in C} D^*(x, A, B)\}$.

Informally, if (X, D^*) is a D^*-metric space, the D^*-metric Hausdorff distance is the greatest of all distances from a point in one set to the closest points in the other two sets plus the distance between those two sets.

The following lemma is an immediate consequence of above Definition 2.2.

Lemma 2.1 Let (X, D^*) be a D^*-metric space. Let $A, B, C \in CB_{D^*}(X)$, then for every $\alpha > 0$ and every $a \in A$, there exist and $b \in B$ and $c \in C$ such that

$D^*(a, b, c) \leq D^*{}_H(A, B, C) + \alpha$.

Definition 2.3 Let (X, D^*) be a D^*-metric space. The set-valued map $T: X \longrightarrow CB_{D^*}(X)$ is said to be a D^*-metric q-set-valued restricted-quasi-contraction if for any $x, y, z \in X$

$$D^*{}_H(Tx, Ty, Tz) \leq q.max\{D^*(x, y, z), D^*(y, Tx, Tx), D^*(z, Tx, Tx), D^*(y, Ty, Tz), D^*(z, Ty, Tz)\}$$

Where $0 \leq q < 1$ and $D^*{}_H$ denotes the Hausdorff metric on $CB_{D^*}(X)$ induced by D^*, that is, for all $A, B, C \in CB_{D^*}(X)$,

$D^*{}_H(A, B, C) = \max\{sup_{x \in A} D^*(x, B, C), sup_{x \in B} D^*(x, C, A), sup_{x \in C} D^*(x, A, B)\}$

Lemma 2.2 Let (X, D^*) be a D^*-metric space and $A \in CB_{D^*}(X)$ and $x \in X$. Then $D^*(x, A, A) = 0$ if and only if $x \in \overline{A} = A$.

Proof: Suppose $D^*(x, A, A) = 0$. This implies, $d(x, A) + d(x, A) + d(A, A) = 0$

Therefore, $d(x, A) = 0$. That is, $\inf\{d(x, y): y \in A\} = 0$. Hence there exist $y \in A$ such that $d(x, y) = 0$, and since $d(x, y) = 0$, we have that $x = y$. Now $y \in A$, implies $x = y \in A$. That is $x \in A$.

Also A is closed since $A \in CB_{D^*}(X)$, which implies that $\overline{A} = A$. Hence, $x \in \overline{A} = A$.

Conversely suppose $x \in \overline{A} = A$, then $x \in A$. This implies, $d(x, A) = 0$.

Therefore, $d(x, A) + d(x, A) + d(A, A) = 0$. Hence, $D^*(x, A, A) = 0$.

■

Lemma 2.3 Let (X, D^*) be a D^*-metric space and $\{x_n\}$ be a sequence in X such that

$D^*(x_{n+1}, x_{n+2}, x_{n+2}) \leq kD^*(x_n, x_{n+1}, x_{n+1})$ for $n = 1, 2, \ldots$ where $0 \leq k < 1$.

Then the sequence $\{x_n\}$ is a Cauchy sequence in X.

Proof: First note that,

$D^*(x_{n+1}, x_{n+2}, x_{n+2}) \leq kD^*(x_n, x_{n+1}, x_{n+1})$

$\leq k^2 D^*(x_{n-1}, x_n, x_n) \leq \cdots \leq k^n D^*(x_0, x_1, x_1)$

Using Definition 1.3 and remark 1.1, for $m > n$,

$D^*(x_n, x_n, x_m) \leq D^*(x_n, x_{n+1}, x_{n+1}) + D^*(x_{n+1}, x_n, x_m)$

$\leq k^{n-1} D^*(x_0, x_1, x_1) + D^*(x_{n+1}, x_{n+2}, x_{n+2}) + D^*(x_{n+2}, x_n, x_m)$

$\leq k^{n-1} D^*(x_0, x_1, x_1) + k^n D^*(x_0, x_1, x_1) + D^*(x_{n+2}, x_{n+3}, x_{n+3}) + D^*(x_{n+2}, x_n, x_m)$

$< (k^{n-1} + k^n + k^{n+1} + \cdots) D^*(x_0, x_1, x_1)$

$= \frac{k^{n-1}}{1-k} D^*(x_0, x_1, x_1)) \longrightarrow 0$as $n \longrightarrow \infty$.

This implies that $\{\{x_n\}\}$ is a Cauchy sequence.

■

The following is our main result, it is a fixed point theorem for D^*-metric set-valued restricted-quasi-contraction maps in a D^*-metric space.

Theorem 2.1 Let (X, D^*) be a complete D^*-metric space and $T: X \longrightarrow CB_{D^*}(X)$be a D^*-metric q-set-valued restricted-quasi-contraction. If $q < \frac{1}{s^2+s}$ where $s \geq 1$ then T has a fixed point in X. That is there exist $u \in X$ such that $u \in Tu$.

Proof: Since T is a D^*-metric q-set valued restricted-quasi-contraction, we have for any $x, y, z \in X$,

$D^*{}_H(Tx, Ty, Tz) \leq q.max\{D^*(x, y, z), D^*(y, Tx, Tx), D^*(z, Tx, Tx), D^*(y, Ty, Tz), D^*(z, Ty, Tz)\}$

Now, $max\{D^*(x, y, z), D^*(y, Tx, Tx), D^*(z, Tx, Tx), D^*(y, Ty, Tz), D^*(z, Ty, Tz)\} = 0$ if and only if $x = y = z$ is a fixed point of T.

We therefore assume,

$max\{D^*(x, y, z), D^*(y, Tx, Tx), D^*(z, Tx, Tx), D^*(y, Ty, Tz), D^*(z, Ty, Tz)\} > 0$f or all $x, y, z \in X$.

Take $\alpha' = \frac{1}{2}\left(\frac{1}{s^2-s} - q\right)$ and $\beta = q + \alpha' = \frac{1}{2}\left(\frac{1}{s^2-s} + q\right)$.

Since we assumed that $q < \frac{1}{s^2+s}$, we have $\alpha' > 0$ and $0 < \beta < 1$.

Let $x_0 \in X$ and $x_1 \in Tx_0$, By Lemma 2.1, there exist $x_2 \in Tx_1$ and $x_3 \in Tx_2$ such that,

$D^*(x_1, x_2, x_3) \leq D^*{}_H(Tx_0, Tx_1, Tx_2) + \alpha$

Also by Lemma 2.1, $D^*(x_1, x_2, x_2) \leq D^*{}_H(Tx_0, Tx_1, Tx_1) + \alpha_1$where $\alpha_1 > 0$.

For $\alpha_1 = \alpha' max \begin{Bmatrix} D^*(x_0, x_1, x_1), D^*(x_1, Tx_0, Tx_0), D^*(x_1, Tx_0, Tx_0), \\ D^*(x_1, Tx_1, Tx_1), D^*(x_1, Tx_1, Tx_1) \end{Bmatrix}$ we have,

$$D^*(x_1, x_2, x_2) \leq D^*{}_H(Tx_0, Tx_1, Tx_1) + \alpha' max \begin{Bmatrix} D^*(x_0, x_1, x_1), D^*(x_1, Tx_0, Tx_0), D^*(x_1, Tx_0, Tx_0), \\ D^*(x_1, Tx_1, Tx_1), D^*(x_1, Tx_1, Tx_1) \end{Bmatrix}$$

$$\leq q.max \begin{Bmatrix} D^*(x_0, x_1, x_1), D^*(x_1, Tx_0, Tx_0), D^*(x_1, Tx_0, Tx_0), \\ D^*(x_1, Tx_1, Tx_1), D^*(x_1, Tx_1, Tx_1) \end{Bmatrix}$$

$$\alpha'.max \begin{Bmatrix} D^*(x_0, x_1, x_1), D^*(x_1, Tx_0, Tx_0), D^*(x_1, Tx_0, Tx_0), \\ D^*(x_1, Tx_1, Tx_1), D^*(x_1, Tx_1, Tx_1) \end{Bmatrix}$$

$$= \beta.max \begin{Bmatrix} D^*(x_0, x_1, x_1), D^*(x_1, Tx_0, Tx_0), D^*(x_1, Tx_0, Tx_0), \\ D^*(x_1, Tx_1, Tx_1), D^*(x_1, Tx_1, Tx_1) \end{Bmatrix}$$

Thus by induction, there exist a sequence $\{x_n\}$ in X such that $x_{n+1} \in Tx_n$ and for all $n \in \mathbb{N}$,,

$$D^*(x_n, x_{n+1}, x_{n+1}) \leq D^*{}_H(Tx_{n-1}, Tx_n, Tx_n) + \alpha' max \begin{Bmatrix} D^*(x_{n-1}, x_n, x_n), D^*(x_n, Tx_{n-1}, Tx_{n-1}), \\ D^*(x_n, Tx_{n-1}, Tx_{n-1}), D^*(x_n, Tx_n, Tx_n) \\ D^*(x_n, Tx_n, Tx_n) \end{Bmatrix}$$

$$\leq \beta.max \begin{Bmatrix} D^*(x_{n-1}, x_n, x_n), D^*(x_n, Tx_{n-1}, Tx_{n-1}), \\ D^*(x_n, Tx_{n-1}, Tx_{n-1}), D^*(x_n, Tx_n, Tx_n) \\ D^*(x_n, Tx_n, Tx_n) \end{Bmatrix}$$

Let$D_n^* = D^*(x_n, x_{n+1}, x_{n+1})$, we then have

$$D_n^* \leq \beta.max \begin{Bmatrix} D^*(x_{n-1}, x_n, x_n), D^*(x_n, Tx_{n-1}, Tx_{n-1}), \\ D^*(x_n, Tx_{n-1}, Tx_{n-1}), D^*(x_n, Tx_n, Tx_n) \\ D^*(x_n, Tx_n, Tx_n) \end{Bmatrix} \tag{2.1}$$

Case I: If $x_n = x_{n+1}$ then $x_n = x_{n+1} \in Tx_n$ is a fixed point of T and the proof will be complete.

Case II: If $x_n \neq x_{n+1}$. Then,

$$max \begin{Bmatrix} D^*(x_{n-1}, x_n, x_n), D^*(x_n, Tx_{n-1}, Tx_{n-1}), \\ D^*(x_n, Tx_{n-1}, Tx_{n-1}), D^*(x_n, Tx_n, Tx_n) \\ D^*(x_n, Tx_n, Tx_n) \end{Bmatrix} \leq max \begin{Bmatrix} D^*(x_{n-1}, x_n, x_n), D^*(x_n, x_n, x_n), \\ D^*(x_n, Tx_n, Tx_n), D^*(x_n, x_{n+1}, x_{n+1}) \\ D^*(x_n, x_{n+1}, x_{n+1}) \end{Bmatrix}$$

$$= max\{D^*(x_{n-1}, x_n, x_n), D^*(x_n, x_{n+1}, x_{n+1})\}.$$

$$= max\{D_{n-1}^*, D_n^*\}.$$

If $max\{D_{n-1}^*, D_n^*\} = D_n^*$, then Equation 2.1 becomes $D_n^* \leq \beta D_n^*$ which contradicts the fact that $0 < \beta < 1$. We therefore conclude that $max\{D_{n-1}^*, D_n^*\} \neq D_n^*$ which implies,

$$max\{D_{n-1}^*, D_n^*\} = D_{n-1}^*$$

Hence Equation 2.1 becomes,

$$D_n^* \leq \beta D_{n-1}^* \tag{2.2}$$

This implies that the conditions of Lemma 2.3 are satisfied, hence $\{x_n\}$ is a Cauchy sequence.

Now since X is complete, there exist $u \in X$ such that $x_n \longrightarrow u$ as $n \longrightarrow \infty$, that is,

$\lim_{n\to\infty} x_n = 0$

We now proceed to show that u is a fixed point of T, that is $u \in Tu$.

Now by Definition 1.3 we have,

$$D^*(Tu,Tu,u) \le D^*(Tu,Tu,x_{n+1}) + D^*(x_{n+1},u,u)$$

$$\le D^*{}_H(Tu,Tu,Tx_n) + \alpha'.max\begin{Bmatrix} D^*(u,u,x_n), D^*(u,Tu,Tu), \\ D^*(x_n,Tu,Tu), D^*(u,Tu,Tx_n) \\ D^*(x_n,Tu,Tx_n) \end{Bmatrix} + D^*(x_{n+1},u,u)$$

$$\le q.max\begin{Bmatrix} D^*(u,u,x_n), D^*(u,Tu,Tu), \\ D^*(x_n,Tu,Tu), D^*(u,Tu,Tx_n) \\ D^*(x_n,Tu,Tx_n) \end{Bmatrix} + \alpha'.max\begin{Bmatrix} D^*(u,u,x_n), D^*(u,Tu,Tu), \\ D^*(x_n,Tu,Tu), D^*(u,Tu,Tx_n) \\ D^*(x_n,Tu,Tx_n) \end{Bmatrix}$$

$$+ D^*(x_{n+1},u,u)$$

Letting $n \longrightarrow \infty$ we get,

$D^*(Tu,Tu,u) \le q.D^*(u,Tu,Tu) + \alpha' D^*(u,Tu,Tu) = (q+\alpha')D^*(u,Tu,Tu) = \beta.D^*(u,Tu,Tu)$That is,

$$D^*(u,Tu,Tu) \le \beta.D^*(u,Tu,Tu) \tag{2.3}$$

Nowsince $0 < \beta < 1$, The only way Equation 2.3 holds is if$D^*(u,Tu,Tu) = 0$.

By lemma 2.2 we get $u \in \overline{Tu} = Tu$, that is $u \in Tu$.

This completes the proof.

■

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